Infinitely Complex Topology Changes with Quaternions and Torsion

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We develop some ideas that can be used to show relationships between quantum state tensors and gravitational metric tensors. After firmly grasping the math by α and Einstein's equation, this is another attempt to shake it and see what goes and what stays. We introduce slightly more rigorous definitions for some familiar objects and find an unexpected connection between the chirological phase Φ^n and the quaternions $\boldsymbol{q} \in \mathbb{H}$. Torsion, the only field in string theory not already present in the theory of infinite complexity, is integrated. We propose a solution to the Ehrenfest paradox and a way to prove the twin primes conjecture. The theory's apparent connections to negative frequency resonant radiation and time reversal symmetry violation are briefly treated.

> "Though the methods are old and the original mathematics literature deeper than the recent physics reworking, what once seemed arcane mathematics has turned out to be a very useful tool for the description of observed strange sets, and winged expressions like 'f of α' have by now become a part of our conceptual vocabulary. "
>
> ∼ P. Cvitanović

The symmetry of the unified field theory is expected to be an amalgamation of four topologies: an orthogonal group SO(3,1), and three unitary groups U(1), $\text{SU}(2)_L$, and SU(3). The special quality of the special subgroups is that the operators in a special group have determinant equal to one. Determinant measures something like volume that can be a positive or negative, real or complex number, in any discrete harmonic of the longitudinal phase Φ^n. Specifically, $\hat{\Phi}$ is the null vector of the group of all hypercomplex rotations of W, where $\mathcal{Z}$ is the probability that a particle will undergo certain dynamics in the presence of a source J.

$$\mathcal{Z}(J) = \mathcal{Z}_0 e^{iW(J)} \tag{1}$$

We want to isolate the dynamic sector $e^{iW(J)}$ and define it as a multiplex with its own dynamical structure unrelated to J. The ensuing relationship is that between chronos and chiros.

For clarity, unitary matrices satisfy $\hat{U}^{-1}\hat{U} = \hat{1}$ but the unitarity of the dynamics is ensured by $\det(\hat{U}) = 1$. The set $\{\hat{\pi}, \hat{\Phi}, \hat{2}, \hat{\imath}\}$ will be labeled ontological to emphasize that the theory of infinite complexity should be considered singular, and only invariant under a group with no operations save $\hat{1}$. The ontological basis is a gateway to new physics because it means states in the past and future need not be invariant under certain operations which are usually elements in physics' set of immutable theoretical underpinnings.

It is not precisely required that the space of state vectors $\mathcal{H}'$ is the same object as the space of dual vectors $\mathcal{H}''$. The most general case is when they form a topological complex in two-to-one correspondence with the Euclidean geometric manifold $\mathcal{H}$.

We will apply new operations to the dual vector alone, leaving the quantum mechanics of state vectors in $\mathcal{H}'$ untouched. This is allowed because the physics of the dual vector is suppressed in the algorithm represented by the $\langle\text{bra}|\text{ket}\rangle$. Philosophically, why should a dual vector be available for computational conjuring when specific creation operators are needed for the original vector? There is no good reason for that.

Looking for new physics in the non-unitary sector, we want matrices with determinant proportional to the numbers in the ontological basis. There are many distinct histories through an algebra in which the volume of some dynamical quantity begins as unity, deforms in a non-volume preserving way, and is then operated upon to produce a local object with volume unity. These objects are $\hat{\pi}$-sites. They relate directly to the Ehrenfest paradox and Einstein's original argument about non-Euclidean geometry.

In the Ehrenfest paradox, length contraction along the circumference of a relativistically spinning disc is coupled with length preservation on its radius to violate π in $C = 2\pi R$. Equation (2) is a specific example of how it is useful to view π as part of a multiplex relating two or more dynamical quantities.

$$C = (R + R)\,\hat{\pi}_0 \tag{2}$$

Just like in equation (1), we can hold one thing constant while scaling the other with Lorentz or other transforms. Is it relevant that we can also do this with the commutator of the Gell-Mann matrices? Consider the following vector in the $(\hat{2} \;\; \hat{\imath})^T$ basis.

$$[\lambda_a, \lambda_b] = f_{abc}\lambda_c \begin{pmatrix} i \\ 2 \end{pmatrix} \tag{3}$$

In any case, the resolution of the Ehrenfest paradox is to crown π with the operator symbol. Then we may solve the system in an operating basis whose null vector is not just in the direction of $\hat{\pi}$, but is $\hat{\pi}$. Equation (2) is also the most rigorous definition of the required map between a diameter and a circumference [1], known from ancient times.

Consider the above in the ontological basis $(\hat{\pi} \;\; \hat{\Phi} \;\; \hat{2} \;\; \hat{\imath})^T$.

$$C = R \begin{pmatrix} 2 \\ 0 \\ \pi \\ 0 \end{pmatrix} \tag{4}$$

$$[\lambda_a, \lambda_b] = f_{abc}\lambda_c \begin{pmatrix} 0 \\ 0 \\ i \\ 2 \end{pmatrix} \tag{5}$$

The plane spanned by chronos and chiros is illustrated in figure 1. Progression from $\aleph$, through $\mathcal{H}$, to Ω marks the direction of increasing chiros and the vertical axis is chronos. Any set of four independent vectors naturally span O(4) and the topology of the ontological basis is also O(3,1) which is great because that is the topology of spacetime. The relationship between the rational real number 2 and the irrational real numbers π and Φ mirrors that between $\hat{J}_x$ and $\hat{J}_y$ in the $\hat{J}_z$ eigenvector basis. Two can be known exactly but there is inherent uncertainty in π and Φ.

Note $\hat{\imath}$ is rigorously unitary and $\hat{2}$ is pseudo-unitary, so they can both contribute to the overall unitary character of the probability interpretation of histories defined in the chronological past in the lower half-plane. $\hat{\Phi}$ and $\hat{\pi}$ share the half-plane of positive chronos and should be associated with non-unitary, turbulent, real dynamics.

If we want the ontological basis to define a quantum theory, its elements better not commute! Of course numbers do commute so we need to consider other pictures. Exponentiated operators can be interpreted as vectors or matrices so we must consider both cases. This consideration defines a new sector because when $\{\hat{\pi}, \hat{\Phi}, \hat{2}, \hat{\imath}\}$ take on purely numerical values as in equations (6-9), the space of all possible operations is not limited to the binary vector-matrix correspondence.

$$\hat{\pi}^2\psi = \pi^2\psi \tag{6}$$
$$\hat{\Phi}^2\psi = \Phi\psi + \psi \tag{7}$$
$$\hat{2}^2\psi = \psi + \psi + \psi + \psi \tag{8}$$
$$\hat{\imath}^2\psi = -\psi \tag{9}$$

The above is mostly unremarkable, but consider the application to increasing entropy. If the action of $\hat{2}$ on ψ is always to split it into two terms, each carrying a qubit, then entropy will increase as $\hat{2}$ is applied. $\hat{\Phi}$ is interesting in this sense because it may not split ψ in a linear application, but only when the operator appears as $\hat{\Phi}^2$. Another possibility is that $\hat{\Phi}^2$ makes a big copy through $\Phi^2 = \Phi + 1$ and $\hat{\Phi}$ makes a smaller copy through $\Phi = 1 + \varphi$.

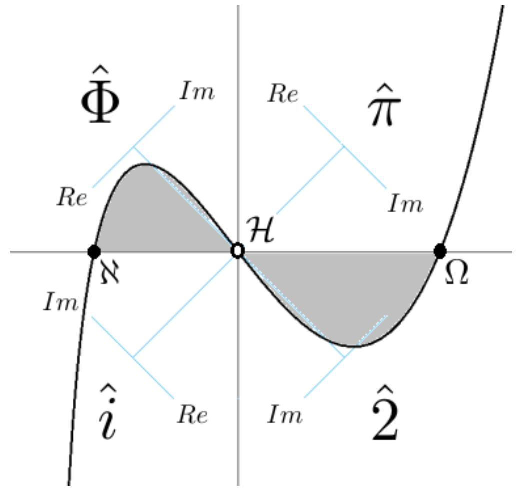

FIG. 1. A lattice is defined on three points where cosmological state tensors are diagonal and torsionless. Given three points, it always possible to calculate a cubic spline function. In turn, the spline can be sampled to gain information about physics in higher than four dimensions. The essence of the interpretation is that $\hat{\pi}$ is local in the first quadrant, $\hat{\Phi}$ is timelike and the null vector of the Hamiltonian, $\hat{2}$ is spacelike and computational, and possibly also a null vector of $\hat{H}$, and $\hat{\imath}$ is imaginary in the mercurial third quadrant. Relating to the new spacelike vector $\hat{2}$, recall that the differential element of the surface of Minkowski space at spacelike infinity is a two-form. [2, 3]

Quantum theory differs from classical in its utilization of complex probability amplitudes on which one must operate with the $\langle \text{bra}|\text{ket}\rangle$ to obtain real numbers. The $\langle \text{bra}|\text{ket}\rangle$ takes complex numbers and returns a purely real number for comparison with experiment. Here, the output of the $\langle \text{bra}|\text{ket}\rangle$ is modified as follows when ψ is a Dirac vector.

$$\text{old} \rightarrow \begin{cases} \psi & : \mathbb{R}^3 \rightarrow \mathbb{H} \\ \langle\psi|\psi\rangle & : \mathbb{H} \rightarrow \mathbb{R} \end{cases} \tag{10}$$

$$\text{new} \rightarrow \begin{cases} \psi & : \mathbb{R}^3 \rightarrow \mathbb{H} \\ \langle\psi|\psi\rangle & : \mathbb{H} \rightarrow \mathbb{R} \otimes \aleph \\ \hat{\pi} & : \mathbb{R} \otimes \aleph \rightarrow \mathbb{R}' \end{cases} \tag{11}$$

Taking a step back, let's reexamine the continuum. To an observer, points representing lattice sites seem like they are adjacent and form a continuum $\mathbb{R}$. Being creative individuals, there is no reason we can't transport the real line to an imaginary space, make all the points discrete, and then define new functions on the possibly continuous set of imaginary points between adjacent elements of $\mathbb{R}$.

Note the density of points in $\mathbb{R}$ does not have to equal that in $\mathbb{R}'$. In fact, it might be that the tensor product with infinity in definition (11) washes out the point density function on $\mathbb{R}$ so that it is no longer recoverable after projection into $\mathbb{R}'$. It is further possible that there is complex hyperstructure encoded on the point density function (which can always be operated upon as a field potential) and that structure manifests observably as the information current. With hyperstructure defined everywhere (via the continuum's point density function) it is possible to have infinite localized point densities and still preserve physics by using a window functional measure that will integrate over exactly $\aleph_0$ little elements dx^0 as if they were in a continuum. This despite the infinitude of other points between them, as in the ϵ-δ proof. We mention this because a varying point density is closely connected to the varying number of points that unilaterally violates conservation of information. Lastly, without infinite point densities there is no infinite self-similarity.

Consider the $\hat{\pi}$ component of some quantity decomposed into figure 1's pictured basis. That component can be further decomposed on the basis, and on and on, implying all real numbers have an imaginary component because $\hat{i}$ is part of the basis. Observables can always be represented by real numbers but what is the imaginary component of an observable quantity? We hypothesize it is each particular observable's interaction with the information current.

$$\psi := \sqrt{i} \qquad \Rightarrow \qquad \mathrm{Im}\langle\psi|\psi\rangle \neq 0 \tag{12}$$

CHRONOS & CHIROS

The TOIC allows for distinct modes in real, continuous time and a discrete psychological time in which man attempts to catch the present moment but never does, thereafter observing quantized phenomena. To clarify usage in this paper, $\hat{\pi}$ relates to the chronological time and $\{\hat{\Phi}, \hat{2}, \hat{i}\}$ relate to the chirological future, present, and past respectively. Consider the identity and how column vectors representing $\{\hat{\Phi}, \hat{2}, \hat{i}\}$ are combined into one tensor state.

$$\psi\,\hat{1} = \left(\frac{1}{4\pi}\psi\right)\hat{\pi} + \left(\frac{\varphi}{4}\psi\right)\hat{\Phi} + \left(\frac{1}{8}\psi\right)\hat{2} - \left(\frac{i}{4}\psi\right)\hat{i} \tag{13}$$

$$\psi_\pi := \frac{1}{4\pi}\psi \tag{14}$$

$$\psi_\Phi := \frac{\varphi}{4}\psi \tag{15}$$

$$\psi_2 := \frac{1}{8}\psi \tag{16}$$

$$\psi_i := -\frac{i}{4}\psi \tag{17}$$

$$\text{Chronos} \quad \rightarrow \qquad \frac{1}{4\pi}\psi(x^\mu) \equiv |\psi;\pi_0\rangle \tag{18}$$

$$|\psi;\Phi\rangle \equiv \begin{pmatrix}\psi_\Phi & 0 & 0\end{pmatrix}^T = \psi_\Phi(x^\mu_+) \tag{19}$$

$$|\psi;2\rangle \equiv \begin{pmatrix}0 & \psi_2 & 0\end{pmatrix}^T = \psi_2(x^\mu) \tag{20}$$

$$|\psi;i\rangle \equiv \begin{pmatrix}0 & 0 & \psi_i\end{pmatrix}^T = \psi_i(x^\mu_-) \tag{21}$$

$$\psi_{ij} = \begin{pmatrix}\psi_\Phi & 0 & 0\\ 0 & \psi_2 & 0\\ 0 & 0 & \psi_i\end{pmatrix} \tag{22}$$

$$\text{Chiros} \quad \rightarrow \qquad \det(\psi_{ij}) \equiv \langle\psi;\pi_1| \tag{23}$$

A dual vector in $\mathcal{H}''$ shall be the output of an operation on a tensor and the tensorial quality of the "dual vector" is a topological complex on the gravitational manifold $\mathcal{H}$. Note equations (19-21) are also the QCD color basis that we want to integrate with gravity and electroweak physics.

A stationary chirological state is one in which the phase is scaled up by Φ in the future and down by φ in the past.

$$\psi_\Phi := e^{i\pi\Phi} \tag{24}$$

$$\psi_2, \psi_\pi := e^{i\pi} \tag{25}$$

$$\psi_i := e^{i\pi\varphi} \tag{26}$$

An intuitive interpretation of the above is that 2 and π are related to transverse phase in the present, and Φ and i define longitudinal phase in the lattice. In the 3D system $\{\hat{\pi}, \hat{\Phi}, \hat{i}\}$ it was sufficient to derive a chronological state from the inner product of two chirological ones [4]. Now there are three chirological states and a triple product is needed.

$$\det(\psi_{ij}) = \psi_\Phi\psi_2\psi_i = e^{2i\pi\Phi} \tag{27}$$

This doesn't look like the dual vector to $e^{i\pi}$ but we will return to that later. Instead, note the determinant of the chirological state tensor is equal to the quaternion rotation of the chronological state by Φ radians.

$$\det(\psi_{ij}) = e^{\boldsymbol{u}\Phi}\psi_\pi e^{-\boldsymbol{u}\Phi} \tag{28}$$

Consider rotations of the half-integer spin vector in the lab. When the observer takes consecutive measurements of the spin, the vector remains stationary. Now let the observer make repeated observations of the same system while the experimental apparatus is rotated in a way such that the spin vector sweeps out a plane. After completing a full lab rotation through 2π radians, the spin vector will only have rotated from zero to π radians. Such is the mystery of spin; the observable lag under rotations is an outstanding example of quaternion mathematics in action.

AN ASIDE

Groups are useful for theory because they make it plain exactly what kind of things can happen to a state in the space in which it lives. It's understood a QCD state can be physically operated upon by the group of SU(3) matrices, and then it is only left up to physicists to find out the amplitudes for different ones. Another example is Lorentz invariance in relativity. It means 4-vectors have to be invariant under 3D rotations and 1D boosts.

A universal fact of rotation matrices is that they rotate about an origin and the main approach to physics has involved only one measly little origin of coordinates. The TOIC has three origins in $\Sigma^\pm$ and $\mathcal{H}$, and this leads to exciting mathematical novelty.

The operator $\hat{M}^3$ rotates a state through $\pi/2$ radians in the complex plane to return the critical value $i\pi\Phi^2$ [5, 6]. The continuous phase of observables should be periodic in $2n\pi$ and it contrasts Φ^n which is discrete and not related to observables. $\hat{M}^{12}$ is required to sweep out the complex plane so through its returned value $\pi^4\Phi^8$, the complex plane spans eight discrete levels of harmonic phase. In reference [7] we suggested the eight phases could generate SU(3) since it has eight generators. However, consider the following.

$$(\hat{\Phi}^2)^2 = \hat{\Phi}^4 \qquad (\hat{\Phi}^2)^3 = \hat{\Phi}^6 \qquad (\hat{\Phi}^2)^4 = \hat{\Phi}^8 \tag{29}$$

$$(\hat{\Phi}^3)^2 = \hat{\Phi}^6 \tag{30}$$

$$(\hat{\Phi}^4)^2 = \hat{\Phi}^8 \tag{31}$$

We present a possible explanation for why QCD is such a difficult theory with interacting gluons while QED and weak nuclear physics are so much more highly utile. Φ^8 is a convenient symmetry but it is not irreducible or "ontological." If the physics of $\hat{M}^{12}$ was truly SU(3), the generators would be independent and not factorizable. True SU(3) is defined on eight prime number harmonics, specifically up to Φ^{13} with $\hat{M}^{20}$.

One case is that Φ^8 is not related to QCD. Another is that the strong force is self-interacting because the QCD group generators seem independent but three of them technically are not.

Consider an application in pure mathematics. The past, present, and future are defined on three sequential integer powers of the golden ratio which are always related by equation (32).

$$\Phi^{N+1} = \Phi^N + \Phi^{N-1} \tag{32}$$

Anomalous contributions to physics in $\hat{\pi}$ (from different powers of Φ representing temporally non-local levels of $\aleph$ [4]) are related to quantum oddities but physics is independent of the absolute phase N. Consider the case when a thorough accounting of all the physics implies that all non-local contributions to physics in the present are scaled as the inverse prime numbers. This is a condition on equation (32) that $N \pm 1$ are always primes separated by an arbitrary integer N. They are twin primes.

If it is possible to use the golden ratio to prove that the spiral lattice structure [4] never collapses because it is perfectly self-similar, and it is shown that there is a harmonic spectrum related to the prime numbers, then that will prove the twin primes conjecture.

Consider twin prime cases of equation (32).

$$\Phi^7 = \Phi^6 + \Phi^5 \tag{33}$$

$$\Phi^{139} = \Phi^{138} + \Phi^{137} \tag{34}$$

$$\Phi^{-5} = \Phi^{-6} + \Phi^{-7} \tag{35}$$

$$\Phi^{-137} = \Phi^{-138} + \Phi^{-139} \tag{36}$$

$$\Phi = 1 + \varphi \tag{37}$$

$$137^{\Phi\pi} = 726... \tag{38}$$

How do prime numbers relate to negative numbers? What is the special case of equation (37)? In the quantum sector, the Fibonacci sequence should be more fundamental than the golden spiral so might this define a kernel on the consecutive ones at the beginning of that – thereafter monotonically increasing – sequence? Could the kernel be the intersection of two spirals [4], one inside the unit circle and one out, as in equations (33) and (35)? Is the 2D unit circle somehow implied by the two unit entries at the beginning of the Fibonacci numbers 1,1,2,3,5,8,13,21...?

TOPOLOGY CHANGE

Consider a general relativistic stress energy tensor T_μ^ν.

$$T_\mu^\nu = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \tag{39}$$

Note pressure p is directly observable while density ρ is not. Mass can be measured on a balance, and volume can be found by optical approximation or fluid displacement, etc. Pressure is directly observable with a barometer but density is always calculated as per Archimedes. T_0^0 evaluates to a negative number so it cannot be *directly* associated with observables or – by proxy of quantum mechanics – Hermitian matrices in Hilbert space.

Hamiltonian physics is nice because once an interested party has any two of a system's dynamical variables, here density and pressure, it is possible to label them p and q and formulate the equations of motion with the Hamiltonian H. Given H, all other physical properties become calculable. Consider the case when an experimenting physicist knows the mass m of an object $\mathcal{O}$. He wants to know if T_μ^ν is right by using it to compute m' and comparing to m.

The topology of the theory can't depend on the shape of the object so we can let $\mathcal{O}$ be spherical and centered at the origin, and still have an instructive exercise. Consider the case of $\rho = \beta r^{-\gamma}$ with $\gamma > 2$.

$$\begin{aligned} V &= \iiint r^2 \sin(\phi) dr d\theta d\phi \\ &= \left(\int_0^\pi \sin(\phi) d\phi\right)\left(\int_0^{2\pi} d\theta\right)\left(\int_0^r r'^2 dr'\right) \end{aligned} \tag{40}$$

$$\begin{aligned} m' &= -\int T_0^0 dV \\ &= \beta \iiint \frac{\sin(\phi)\, d\theta d\phi dr''}{|\vec{r'} - \vec{r''}|^{\gamma-2}} \end{aligned} \tag{41}$$

Note V takes the form of a triple product. Before continuing with m', consider the chirological triple product modeled on equation (40). The definite integral form exhibits further behavior.

$$\begin{aligned} V' &= \left(\int \psi_\Phi d\pi\right)\left(\int \psi_2 d\pi\right)\left(\int \psi_i d\pi\right) \\ &= ie^{2i\pi\Phi} \end{aligned} \tag{42}$$

The only effect of this – possibly meaningless – formulation is a complex phase shift by $\pi/2$ but we digress.

To compute m' one must choose the origin of the double primed integration variables away from the center of $\mathcal{O}$ else the integral will explode due to $|\vec{r'} - \vec{r''}|$. Even when that origin is far from $\mathcal{O}$, the arrangement is only a kludge because the equations of motion still have to work if $\mathcal{O}$ moves to the double primed origin. The theory varies from place to place between real and undefined so it is not invariant under translations. The defect is hard coded into the topology and it is where the physics that everyone already knows is connected to new physics. Much like a diverging diamond is topologically equivalent to a roundabout, or the standard example of a coffee mug always having a hole in it, there is always a part of the theory that can't be computed.

Consider that workhorse of modern physics, ye olde photon propagator and the odious factor $i\epsilon$ that has resolutely no business in the denominator (other than that it generates predictions that agree with experiment.) The sudden appearance of the dual vector for the $\langle \text{bra}|\text{ket}\rangle$ is dubious but that word is not adequate to describe $i\epsilon$.

$$\frac{-ig_{\mu\nu}}{\vec{k}^2 + i\epsilon} = \frac{-i}{\vec{k}^2 + i\epsilon}\begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{43}$$

The term is inserted to smudge over the topological defect in the Hamiltonian space. Otherwise physics will explode in the reference frame where $\vec{k}$ is zero. Physics must never explode!

The connection to the TOIC "big picture" is clear when the Feynman propagator is one of two possible combinations of the advanced and retarded propagators that sidestep the defect in the upper and lower complex half-planes.

Quantum models of topology change, such as those needed to move the defect around, are well developed and can almost always be reduced to changing boundary conditions on Hilbert space [8–10]. The approach here is to begin with standard laboratory observables and then develop topology change in a way that never disrupts the useful, real output of the historical theory. Specifically we seek to complexify the global topology by introducing multiplex-valued boundary conditions on Hilbert space. We introduce the information current by assigning different BCs to $\mathcal{H}'$ and $\mathcal{H}''$.

Consider the following congruence.

$$\begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} \cong \begin{pmatrix} e^{i\theta_x} & ie^{i\theta_z} \\ ie^{-i\theta_z} & e^{i\theta_y} \end{pmatrix} \tag{44}$$

As a working definition we can say orthogonal matrices are diagonalizable and have parameter elements, and unitary matrices have exponentiated parameter elements (satisfying $\hat{U}^{-1}\hat{U} = \hat{1}$). As an example, consider the map from Minkowski space to modified spacetime where time is wrapped around a cylinder.

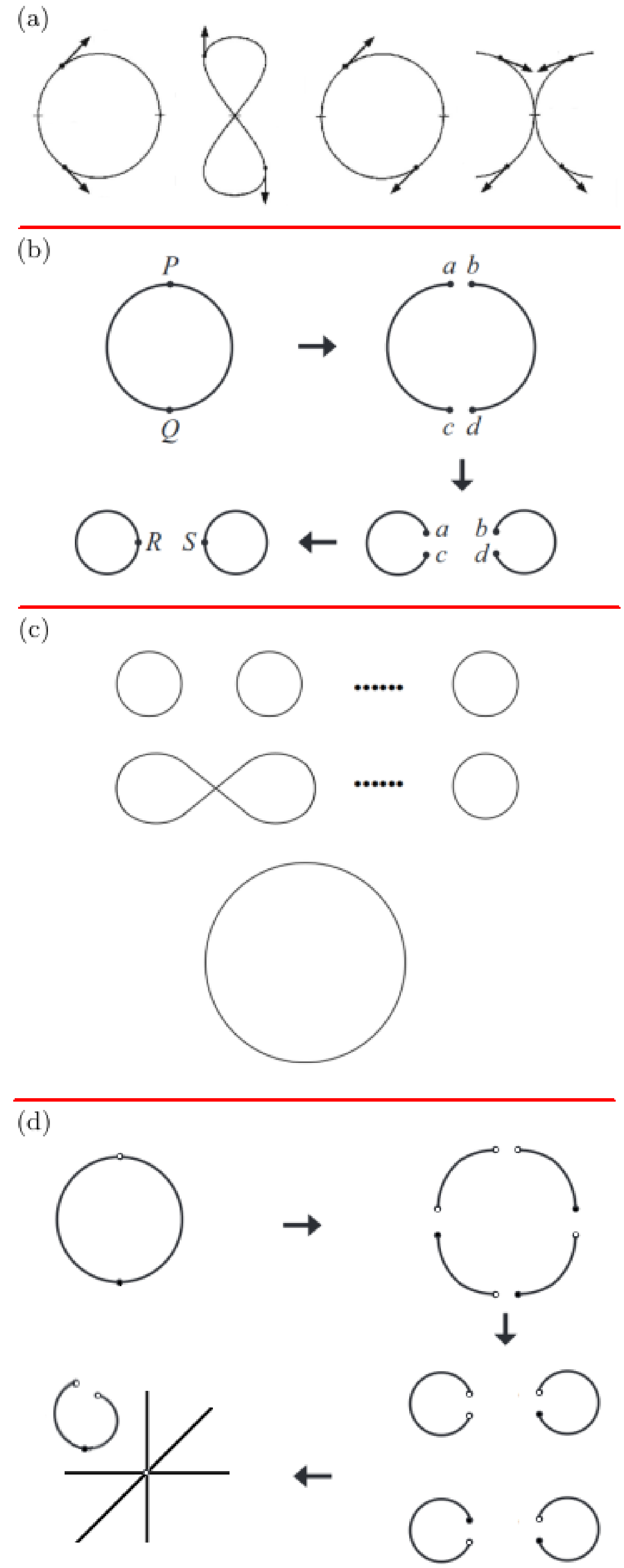

FIG. 2. Four models of topology change. On top, the model that motivated the original acronym MCM in references [1, 11]. The next two are variations on the same model of topology change from references [8] and [9] respectively. Fourth is the model of topology change suggested here.

$$\begin{pmatrix} -c^2 t & 0 \\ 0 & x^i \end{pmatrix} \mapsto \begin{pmatrix} e^{-ic^2\theta_t} & 0 \\ 0 & x^i \end{pmatrix} \tag{45}$$

This is pictured in figure 2a which first appeared in reference [11]. As the simplest conceivable model of topology change, it is no surprise that the same demonstrative example appears in references [8] and [9], shown here as figures 2b and 2c respectively. The model in 2a was complete in reference [11] but it wasn't until reference [1] that we specifically identified the state in the present. Such was in response to one of "the most important questions any quantum cosmology theory should address" listed in reference [12]: "Can we extract, from the arguments of the wavefunction, one variable which can serve as *emergent time* with respect to which the other arguments 'evolve'?"

The answer is yes. It is possible to extract that as the superposition of the positive and negative time components [1]. This is the same mechanism by which quantum computing is expected to replace conventional, binary-bit computing.

Consider an operational difference between figures 2a and 2b. Where they define boundary conditions on the endpoints, the TOIC considers a wavepacket moving with a certain wave vector, far from the endpoints. Those points represent the earliest past of existence and its deepest future [1, 11], so things that are local to the endpoints probably don't affect the wavepacket. Said another way, the second model's circumference is of an indeterminate length (probably less than one meter) where 2a has a circumference at least on the order of dozens of billions of light years. Since the endpoints are not local to the wave, we are motivated to move the topological defect to the endpoints of the timelike interval in the final phase of figure 2d. When the endpoints are at past and future infinity, we have a theory that is locally invariant under all finite translations.

A point from time is spliced into the null point at the origin of space, and the source and sink of the information current are moved to the the ends of time. We have already shown that the topology of the quantum phase U(1) should be modified to include a null point [3, 4] and that motivates the one in the circle at the beginning of figure 2d. Without it, there wouldn't be a unique interval with two null endpoints allowing us to specify one of the four segments as temporal.

On the null point, consider congruence (44). The Cartesian coordinates $\{x, y, z\}$ are not periodic but $\{\theta_x, \theta_y, \theta_z\}$ are. Analytic functions are single-valued, one-to-one maps between a domain and a range so when the time axis is wrapped around a cylinder, to preserve analyticity, and therefore physicality, the interval must be scaled so that it is not periodic on the cylinder. This is an important point and we will return to it below.

Figure 2a has the entire real line encoded on it twice, once for each universe [1, 11]. The topology change that was the foundation of the MCM and this entire course of research is a map from the infinite to the finite $\mathbb{R} \mapsto \pi$. This is the standard conformal rescaling that is so prevalent in Penrose diagrams and many other places in physics. It also describes the new map $\mathbb{R} \otimes \aleph \mapsto \mathbb{R}'$.

Consider the structure of the higher dimensional spaces where we let chiros be timelike in Σ^+.

$$g^{\pm}_{AB}\Big|_{\aleph,\Omega} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & \mp\Phi^{\pm1} \end{pmatrix} = \begin{pmatrix} -c^2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \mp\Phi^{\pm1} \end{pmatrix} \tag{46}$$

After accounting for general relativity in four dimensions, the 5D tensor has eight components left which look like a vector and a dual vector.

$$g^{\pm}_{AB} = \begin{pmatrix} g_{\mu\nu} & g^{\mu}_{\pm} \\ g^{\pm}_{\nu} & \mp\Phi^{\pm1} \end{pmatrix} = \begin{pmatrix} -c^2 & 0 & 0 & 0 & g^0_{\pm} \\ 0 & 1 & 0 & 0 & g^1_{\pm} \\ 0 & 0 & 1 & 0 & g^2_{\pm} \\ 0 & 0 & 0 & 1 & g^3_{\pm} \\ g^{\pm}_0 & g^{\pm}_1 & g^{\pm}_2 & g^{\pm}_3 & \mp\Phi^{\pm1} \end{pmatrix} \tag{47}$$

Recall that the off-diagonal components of $g^{\pm}_{AB}$ need not be zero away from the lattice sites. The object considered here is constructed for simplified analysis of the fifth dimension. In general, the 4D upper-left space will contain ten arbitrary symmetric components and six equally arbitrary anti-symmetric components.

To make each quadrant its own self-similar space as in figure 1, a 9D matrix is needed.

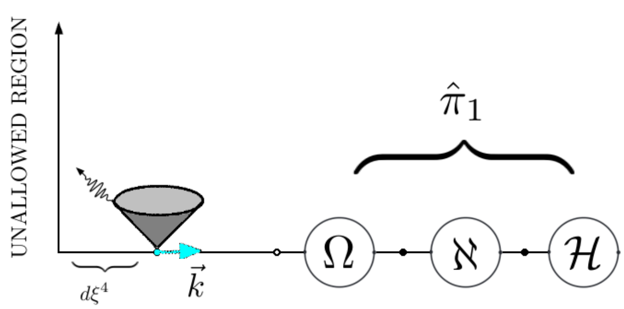

FIG. 3. The $\hat{\pi}_0$-site refers to the region $(-\pi/2, \pi/2)$ and $\hat{\pi}_1 \in (\pi/2, 3\pi/2)$. The object moving to the right on the chirological worldline must be moving faster than the speed of light else its future light cone will pierce the topological obstruction at $\xi^4 = 0$.

$$g' = \begin{pmatrix} -c^2 & 0 & 0 & 0 & g^0_+ & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & g^1_+ & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & g^2_+ & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & g^3_+ & 0 & 0 & 0 & 0 \\ g^+_0 & g^+_1 & g^+_2 & g^+_3 & \mp\Phi^{\pm1} & g^-_5 & g^-_6 & g^-_7 & g^-_8 \\ 0 & 0 & 0 & 0 & g^5_- & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g^6_- & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g^7_- & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g^8_- & 0 & 0 & 0 & 0 \end{pmatrix} \tag{48}$$

Where the Dirac matrices can be represented as 2D matrices with Pauli matrix elements, we find a similar opportunity for dimensional reduction between a 9D object like g' and the 3D rotation matrices. Also note: a matrix whose elements are themselves matrices is almost the definition of a fractal matrix.

The $so(3)$ algebra of rotations must define the topology of the x^i subspace of string theory's 10D space. The remaining six spatial dimensions $x^i_{\pm}$ are encoded with space in the past and future so they must also independently share the SO(3) topology. Therefore we should define the 6D space as SO(3)⊗SO(3). Furthermore, the 11D space of M-theory appears conveniently as $\mathbb{R}_t \otimes \mathbb{R}_\xi \otimes \mathbb{R}^3_{\mathcal{H}} \otimes \mathbb{R}^3_{\Omega} \otimes \mathbb{R}^3_{\aleph}$. The TOIC diverges from M-theory in the topological obstruction between $\mathbb{R}$ and $\mathbb{R}'$.

The algebra of quaternions $\mathbb{H}$ can be decomposed into left and right rotational subgroups S^3_R and S^3_L. Let us include only left isoclinic rotations to give the chirality required by the standard model. Non-unitary operations in Σ^+ will be scaled with respect to the same operations in Σ^- implying an added shear component when both are considered simultaneously. That will create a shear plane where $\Sigma^{\pm}$ approach $\mathcal{H}$ at $\xi^4 = 0$. The shear plane is immediately identifiable as the topological obstruction that is the TOIC's new boundary condition. It also tells us about physics in higher dimensions.

Let something go into the future by leaving the shear plane the positive ξ^4-direction as in figure 3. If the velocity is less than c, the future light cone will pierce the topological obstruction, meaning it is not actually an obstruction at all. If it is moving at c, the edge of the light cone will be on the boundary of the unallowed region, and that is also not allowed because $\Sigma^{\pm}$ do not include the boundary at $\xi^4 = 0$ [3]. Therefore, granted many assumptions, we may conclude that only tachyons propagate on ξ^4. Also, ξ^4 is periodic so we can define its topology as the broken U(1) interval with the chronological worldline going through the null point in the direction perpendicular to the plane of the circle.

Consider the connection to spin. Arguments against spin arising from actual mechanical rotations rely on the expected rate of rotation being greater than the speed of light but now that is allowed. Reference [13] states that even without reliance on superluminal rotations, spin still has a classical analog in the angular momentum of a circularly polarized wave. The author describes earlier work by Belinfante and others showing that the electron's spin and magnetic moment can arise from a "circulating flow of energy" and a "circulating flow of charge" in the electron's wave field. All of this strongly agrees with the TOIC spin mechanism which also states that spin must have a classical analog [14].

On the rarefied plane $\mathcal{H}$, consider the following from reference [10].

> "There are indications from theoretical considerations that spatial topology in quantum gravity can not be a time-invariant attribute, and that its transmutations must be permitted in any eventual theory. [*sic*] The best evidence for the necessary topology change comes from the examination of the spin-statistics theorem for the so-called geons. Geons are solitonic excitations caused by twists in spatial topology."

Note the connection to the spin-statistics theorem. It has been argued that the TOIC prediction for new spin-1 particles [14] has already been ruled out due to the Landau-Yang theorem which depends heavily on the spin-statistics theorem. It is noteworthy that the geon, which here describes the universe and therefore all observables, is a special case of spin-statistics.

QUATERNION ROTATION

This section leaves many unanswered questions due to its narrow scope. We focus on how quantum mechanics' ordinary non-abelian rotation dependency in three dimensions can be morphed into a 4D hypercomplex quaternion dependency. Below we will refer to a possible modification of the group of quaternions as $\mathbb{H}'$. However, it may be that $\mathbb{H}'$ does not exist and the ordinary quaternions $\mathbb{H}$, being inseparable from 4D rotation, will perform all the requisite operations.

Consider a quaternion $\boldsymbol{q}$.

$$\boldsymbol{q} = a\mathbf{1} + b\boldsymbol{i} + c\boldsymbol{j} + d\boldsymbol{k} \tag{49}$$

x	**1**	***i***	***j***	***k***
1	1	i	j	k
i	i	-1	k	-j
j	j	-k	-1	i
k	k	j	-i	-1

(50)

We want to study how this relates to the following, which may or may not be a quaternion. Consider $\boldsymbol{\psi}_q$ and an arbitrary example algebra.

$$\boldsymbol{\psi}_q = \psi_\pi \boldsymbol{\pi} + \psi_\Phi \boldsymbol{\Phi} + \psi_2 \mathbf{2} - \psi_i \boldsymbol{i} \tag{51}$$

x	**2**	$\boldsymbol{i}$	$\boldsymbol{\Phi}$	$\boldsymbol{\pi}$
2	$\hat{2}+\hat{2}$	$\hat{i}\hat{2}$	$\hat{\Phi}\hat{2}$	$\hat{\pi}\hat{2}$
$\boldsymbol{i}$	$\hat{i}+\hat{i}$	$\hat{i}^2$	$-\hat{\Phi}\hat{\pi}$	$\hat{\pi}\hat{i}$
$\boldsymbol{\Phi}$	$\hat{\Phi}+\hat{\Phi}$	$\hat{i}\hat{\Phi}$	$\hat{\Phi}^2$	$-\hat{\pi}\hat{\Phi}$
$\boldsymbol{\pi}$	$\hat{\pi}+\hat{\pi}$	$-\hat{i}\hat{\pi}$	$\hat{\Phi}\hat{\pi}$	$\hat{\pi}^2$

(52)

A unit quaternion has real coefficients so ψ_i needs special accommodation in $\mathbb{H}'$. There are three operations on quaternions: addition, multiplication, and quaternion multiplication. The quaternion product of two elements of $\mathbb{H}$ depends on the choice of basis so it might be possible to choose a "second quantized" basis such that $\boldsymbol{i} := (0\,\sqrt{i}\,0\,0)^T$. In that case all the $\hat{i}$'s in table (52) need to be modified and notably the $\boldsymbol{ii}$ component becomes just $\hat{i}$.

In $\mathbb{H}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are taken to be some square roots of -1 other than i so what is their relationship to $\boldsymbol{\pi}$ and $\boldsymbol{\Phi}$? π and Φ are themselves incalculable, so what is the number that is their square? As demonstrated above, it is indisputably possible to write down an algebra relating $i^2 \cong j^2 \cong k^2 = -1$ to $\sqrt{i}^2 \cong \Phi^2 \cong \pi^2 = i$. Can irrationality be substituted for imaginarity? It is impossible to digitize an irrational number one time much less twice as needed for multiplication so it is not unreasonable to define the square as imaginary.

Consider the investment-grade isomorphism between $\sqrt{i}$ and $\hat{\Phi}^8$. The quantum mechanical phase is periodic in i^4 so $\mathbb{C}^2$ spans eight longitudinal phases *and* also happens to be periodic in $(\sqrt{i})^8$.

We want to generalize QM to 4D quaternion rotations in the O(4) space spanned by the ontological basis. To that end, consider a 3D rotation through θ radians about the $\hat{n}$-direction.

$$\hat{\mathcal{U}} = e^{-i\theta\hat{n}\cdot\hat{J}^i} \tag{53}$$

$$\psi_3' = \hat{\mathcal{U}}\psi \tag{54}$$

Quaternions are associated with rotations in O(3) when the identity operator is stripped of its operator properties in a transmogrified quaternion "equation."

$$\begin{aligned} \boldsymbol{q}_? &= a + b\hat{i} + c\hat{j} + d\hat{k} \\ &= v_0 + \vec{v} \end{aligned} \tag{55}$$

This is very interesting for a few reasons. First is that one should not add a scalar and a vector. Second, it motivates the algebra of the 3-vector cross product which is easily verified as a subspace of $\mathbb{H}$. Third, it contradicts our definition of the Euclidean unit vectors as renamed versions of $\hat{1}$ [6].

The hatted vectors in equation (55) are written that way to show the application to 3D rotations but they are still quaternions: $\boldsymbol{i} \equiv \hat{\imath}$, $\boldsymbol{j} \equiv \hat{\jmath}$, $\boldsymbol{k} \equiv \hat{k}$. Such numbers are like the imaginary number with a hat on it, not the number one with a hat on it. Therefore, redefining a scalar as a vector pointing along an unspecified number line may not be a universal identity operation. However, the following identities sufficiently connect $\hat{\imath}$ and $\hat{1}$ so that the specifics can be set aside for now.

$$|\boldsymbol{i}| = \sqrt{i} \qquad |\hat{\imath}| = i \qquad |i| = 1 \tag{56}$$

In reference [7] we introduced the operator $\hat{\Upsilon} \equiv \hat{U} + \hat{M}^3$ and instructed the reader to ignore the difficulty associated with adding a tensor operator $\hat{M}^3$ to the vector operator $\hat{U}$. The following is a better definition.

$$\hat{\Upsilon}\psi = \frac{1}{4\pi}|\psi;\pi\rangle + det(\psi_{ij}) \tag{57}$$

$$\hat{U}\psi \cong \vec{\psi} \cdot \hat{\pi} \tag{58}$$

$$\hat{M}^3\psi \cong \det(\psi_{ij}) \tag{59}$$

It may prove useful to define the following tensor-vector sum as a rank-two quaternion.

$$\boldsymbol{\psi}'_q = \psi + \psi_{ij} \tag{60}$$

This doesn't look like a quaternion or an octonion and that implies a possible new normed algebra $\mathbb{H}'$.

Consider the exponent in equation (53). The imaginary number is present, a parameter θ, a vector of generators $\hat{J}^i$, and the null vector $\hat{n}$ that selects from $\hat{J}^i$. This scales up to 4D by taking the full quaternion rotation from the left and right into account.

$$\hat{\mathcal{Q}}_L = e^{\boldsymbol{u}\boldsymbol{\theta}} \tag{61}$$

$$\hat{\mathcal{Q}}_R = e^{-\boldsymbol{u}\boldsymbol{\theta}} \tag{62}$$

$$\psi'_4 = \hat{\mathcal{Q}}_L \psi \hat{\mathcal{Q}}_R \tag{63}$$

The unit quaternion $\boldsymbol{u}$ plays the role of both the imaginary number and the generator in the 3D rotation operator. Since the 3D generator is the product of $\hat{J}^i$ and $\hat{n}$, it should also be possible to define the generator of quaternion rotations as the output of the product of a null quaternion and a quaternion of generators. Let the hat denote the null object.

$$\hat{\mathcal{Q}}_L = e^{\hat{\boldsymbol{u}}_1 \cdot \boldsymbol{u}_2 \theta} \tag{64}$$

Permutations on $\boldsymbol{u}$ as the generator and $\boldsymbol{u}$ as the null vector are a convenient source of fractal structure. One possible application is to say that in the $\hat{\pi}$ basis, the null vector is on the left and in the $\hat{\Phi}$ basis it is on the right. This is not a far stretch because we already have behavior whereby a vector's assigned manifold is specified by the ontological basis.

$$\hat{\pi} : \hat{\mathcal{Q}}_L = e^{\hat{\boldsymbol{u}}_1 \cdot \boldsymbol{u}_2 \theta} \tag{65}$$

$$\hat{\Phi} : \hat{\mathcal{Q}}_L = e^{\boldsymbol{u}_1 \cdot \hat{\boldsymbol{u}}_2 \theta} \tag{66}$$

It should be further possible to define asymmetric "quaternion rotations" involving four unit quaternions with two in $\hat{\mathcal{Q}}_L$, and two in $\hat{\mathcal{Q}}_R$. Each $\boldsymbol{u}$ has four independent components, so there are sixteen elements contributing to the following operation.

$$\psi'_4 = e^{\boldsymbol{u}_1 \cdot \boldsymbol{u}_2 \theta} \psi e^{-\boldsymbol{u}_3 \cdot \boldsymbol{u}_4 \theta} \tag{67}$$

On a related note, the Dirac vector is a 4-vector, and we add a multiplex to each component so the term $\bar{\psi}\psi$ has sixteen independent components. A fractal multiplex on ψ means there are actually infinity independent components, but we can consider a "first tier" of the sixteen discussed here. Different scalar, vector, tensor, and possibly other states and/or operations can be defined on different permutations of the multiplectic algebra.

The golden ratio entered the study to motivate parity violation [1] and the physics described here strongly reinforces that idea. The amplitudes for parity violating processes are small because the Φ component in the future and the $-\varphi$ component in the past don't quite sum to one. In a perfect world it would always be true that $\Phi - \varphi = 1$ but here everything has an unpaired term at infinity which is a qubit of unspecified topology, depending on the term's history. An irrational residue filtering through the dynamics as the information current gives $\Phi - \varphi = 1 + d\aleph$. In this picture, the cross-sections for parity violating processes in colliders measure the typical asymmetry between the qubits at past and future computational infinity.

Consider a general parity violating matrix where c_1 is the cosine of the first variable etc.

$$K = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & [c_1 c_2 c_3 - s_2 s_3 e^{i\delta}] & [c_1 c_2 s_3 + s_2 c_3 e^{i\delta}] \\ s_1 s_2 & [c_1 s_2 c_3 - c_2 s_3 e^{i\delta}] & [c_1 s_2 s_3 - c_2 c_3 e^{i\delta}] \end{pmatrix} \tag{68}$$

The non-parity violating version of K would have either a sine or a cosine everywhere $e^{i\delta}$ appears. Parity violation must therefore be associated with the topological defect in the Euler formula [4]. Note K makes use of that defect four times, and not two, as might be intuitively associated with parity.

Consider the sixteen dynamical avenues embedded in $g^{\pm}_{AB}$. By defining δ as a multiplex, each distinct pairing of elements from g^{μ} and g_{ν} can define a unique operation, K being just one familiar example.

$$g^{\pm}_{AB} = \begin{pmatrix} -c^2 & 0 & 0 & 0 & g^0_{\pm}e^{i\delta_1} \\ 0 & 1 & 0 & 0 & g^1_{\pm}e^{i\delta_2} \\ 0 & 0 & 1 & 0 & g^2_{\pm}e^{i\delta_3} \\ 0 & 0 & 0 & 1 & g^3_{\pm}e^{i\delta_4} \\ g^{\pm}_0 e^{i\delta_5} & g^{\pm}_1 e^{i\delta_6} & g^{\pm}_2 e^{i\delta_7} & g^{\pm}_3 e^{i\delta_8} & \Phi \end{pmatrix} \tag{69}$$

This structure is clearer than $\bar{\psi}\psi$ because we are already dealing with four different 4-vectors $\{g^{\mu}_{+}, g^{\mu}_{-}, g^{+}_{\nu}, g^{-}_{\nu}\}$. Just as the surfaces of constant curvature in the cosmological unit cell connected Kaluza's cylinder condition [3], we find a similarly natural connection to the sixteen component Clifford algebra.

KALUZA KLEIN COMPACTIFICATION

One of the apparent failures of Kaluza theory is that the field equations deduced from Kaluza's 5D formulation imply the electromagnetic field strength tensor $F_{\mu\nu}$ is zero everywhere [15].

$$\begin{aligned} F_{\mu\nu} &= (\partial_\mu A_\nu - \partial_\nu A_\mu) \\ &= \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix} \end{aligned} \tag{70}$$

$F_{\mu\nu}$ is topologically equivalent to the tensor product of a vector and a dual vector so we are presented with an intuitive resolution. Use a pair of $|g^{\mu}_{\pm}\rangle$ and $\langle g^{\pm}_{\nu}|$ so that the field equations are satisfied, and then use the other pair to define a field with non-zero strength.

The TOIC ansatz for cosmological arrangement puts the origin of ξ^4 in $\mathcal{H}$ so its value at the origin of $\Sigma^{\pm}$ is $\pm\infty$ [3]. This implies that the 5D origin of $\xi^A_{\pm}$, usually just a point, is a vector connecting the center of $\xi^{\mu}_{\pm}$ with that of ξ^4. The magnitude of that vector should define the dimensional transposing parameter in the piecewise, point-like definition of the origins in $\Sigma^{\pm}$. We hypothesize the parameter is proportional to the golden ratio so the magnitude of the vector should be Φ. This must be the vector $\hat{\Phi}$ that we were already studying.

As pointed out in reference [15], "Klein showed in 1926 that Kaluza's cylinder condition would arise naturally if the fifth coordinate had (1) a circular topology, in which case physical fields would depend on it only periodically, and could be Fourier expanded; and (2) a small enough ("compactified") scale in which case the energies of all Fourier modes above the ground state could be made so high as to be unobservable."

Here the cylinder condition also arises naturally – though to markedly less fanfare – when (1) the fifth coordinate has golden spiral topology and (2) a topological obstruction prevents any periodic dependence of the field on the compactified dimension(s). Periodicity is not implied by a spiral but spiral dynamics can still be projected onto on a circle when the longitudinal phase Φ^n acts as a winding number on the unbroken U(1) topology. Therefore, it should not be necessary to look beyond the standard methods of KK compactification to deal with the unobservable dimensions of the modified cosmological model.

Each universe in figure 2a is confined to an interval of π radians, as marked by the horizontal hashes. We will interpret this as a constraint on the phase and not on distance. To implement that constraint, do a conformal rescaling of all the dynamical entities in a system so that nothing has its transverse phase leave the interval defined on $(-\pi/2, \pi/2)$ radians. This is nothing but an ordinary gauge transformation.

$$\psi(x) \mapsto e^{i\beta(x)}\psi(x) \tag{71}$$

Just throw some capriciously large numbers in there and you can easily guarantee the satisfaction of the requirement during some finite computational run.

To get to the next moment $\hat{\pi}_1$, the phase needs to advance by at least $\pi/2$ and quaternion rotation by $\pi/2$ will give a multiplicative factor π just like what is required for equation (72).

$$\langle\psi_F; \hat{\pi}_1|e^{-i\hat{H}t}|\psi_I; \hat{\pi}_0\rangle = 1 \tag{72}$$

$$\langle\psi_F; \hat{\pi}_0|e^{-i\hat{H}t}|\psi_I; \hat{\pi}_0\rangle = 0 \tag{73}$$

The amplitude for the particle to stay in the same moment is zero. It always has to go the next moment $\hat{\pi}_1$. Recall $(\hat{\pi})^n \equiv \hat{\pi}_n$ but π^0 and π^1 are the only two powers that seem relevant. This is due to the ontological gauge constantly rescaling back to $\theta = 0$ before the phase can advance to the region defined as $\hat{\pi}_1$.

The observer is always at $t = 0$ and the associated phase $\omega t + \delta$ is also zero. When transitioning back into the $\hat{\pi}$ basis of the cognitive frame after any computational operation there must be a conformal rescaling back to zero. To preserve the relative phase of all qubits everywhere, this rescaling washes through all space. No matter how much time goes by between successive comparisons of theory to experiment, due to the above mentioned large numbers, the phase is guaranteed never to advance by more than π. This is a topological obstruction in the phase space of the phase.

Figure 2d shows that the time axis has two null endpoints and length $\pi/2$ implying the absolute value of the phase being confined to $[0, \pi/2)$ instead of $(-\pi/2, \pi/2)$, but no matter. A point from time is chosen to complete the spatial topology at the origin where we have moved a spatial null point. Where did the other two spatial null points go? Do they form a null multiplex or is this, again, related to uncertainty in $\hat{J}_x$ and $\hat{J}_y$ in the $\hat{J}_z$ eigenvector basis? This will be an unresolved issue going forward.

What is clear is that time's null endpoints in the original model of topology change (figure 2a) are separated by π. The observer is always moving toward the future so it makes sense that the actual interval may be restricted to $\pi/2$ because the phase between zero and $-\pi/2$ is identified with the past. This discrepancy may also be related to the fact that SU(2) is defined on the interval between zero and 4π rather than that between zero and 2π. If the allowed interval is reduced by half, the associated angle of rotation should also be reduced by half to $\pi/4$.

Consider Hawking radiation in the context of topological obstructions. Near the horizon there is a quantum fluctuation and one particle starts falling inward before it annihilates with its partner. Once its trajectory pierces the horizon, its phase space is spontaneously truncated so that no future trajectory ever leaves the interior of the event horizon. (In reference [16] we showed how a fractal embedding of event horizons in a charged, rotating black hole is a good descriptor for cosmological lattice translations.)

The radius of the black hole is proportional to the mass enclosed so it has respective radii r_{out} and r_{in} before and after the particle falls behind the horizon. When is the moment that the particle's phase space changes? It can't change until the particle passes the horizon, and when it does the black hole's radius has already changed to r_{in} meaning the particle is inside by more than a differential element of distance. Hence the moment we are examining can no longer be the moment the radius changed. This is an unresolved paradox.

To derive Hawking radiation, it is necessary to advance a trajectory from r_{far} to r_{close}. When the trajectory is very close, all the relevant information is exported to a parameter file. The information is injected inside the black hole and then someone starts the stopwatch running again. With one particle inside, the other escapes as Hawking radiation despite there being no physical trajectory through the horizon.

Consider the operation of exporting to a parameter file. $\hat{2}$ may create a computational state entangled with the physical state via $2 \mapsto 1 + 1$. Let the entanglement symmetry of $\hat{2}$ guarantee that the topologies of the residue on each term are identical while they are entangled.

Starting with the most minimal representation of U(1), consider the identity map to $\hat{2}$.

$$\psi \mapsto \frac{1}{2}\psi\hat{2} \tag{74}$$

$$\psi_\Phi := \frac{1}{2}e^{i\Phi} \tag{75}$$

$$\psi_2 := \frac{1}{2}e^{i} \tag{76}$$

$$\psi_i := \frac{1}{2}e^{i\varphi} \tag{77}$$

$$\det(\psi_{ij}) = \frac{1}{8}e^{2i\Phi} \tag{78}$$

This looks just like the ψ_2 term in equation (13). Now add to that a quaternion rotation by $\pi/4$.

$$\hat{\mathcal{Q}}_L \det(\psi_{ij})\hat{\mathcal{Q}}_R = \frac{1}{8}e^{i\pi\Phi} \tag{79}$$

This is supposed to be the future dual vector $\langle\psi;\pi_1|$ to $\psi_\pi := e^{i\pi}$ so it is good that the 2 no longer appears in the exponent. The 1/8 pre-factor may indicate that this is only valid in the $\hat{2}$ computational basis, but not valid for other components. The negative sign required for the dual vector to be the complex conjugate of the vector is missing but it can be inserted almost anywhere without disrupting the structure of the theory.

Are these just random mathematical artifacts? What are these operations and why should quaternion rotations be related to anything? For a satisfying answer consider $\hat{M}^3$ and its interpretation as a succession of quaternion rotations about the different origins of $\{\aleph, \mathcal{H}, \Omega\}$.

> "The flow of time proceeds as a quantum clockwork. With the application of the evolution operator $\hat{M}$, the observer's connection to $\mathcal{H}_i$ is released and reconnected to $\aleph_{i+1}$. $\hat{M}$ is applied again breaking the connection to Ω_i. That end of the observer function is reconnected to $\mathcal{H}_{i+1}$ then a third application of $\hat{M}$ restores the original arrangement with a connection between $\mathcal{H}_{i+1}$ and Ω_{i+1}." [5]

TORSION

Operating under the assumption, as we have been, that string theory is right, there must be a torsion field complementing the gravitational and scalar fields derived in references [5] and [3]. Since torsion is just the antisymmetric part of the affine connection, there is no modification required to include that third-rank tensor in the TOIC. Simply require that the cosmological lattice points are defined where the strength of the torsion field is zero. Since geometry with torsion has an inherent handedness, and the torsion only exists in $\Sigma^{\pm}$, but not $\mathcal{H}$, it is perfectly applicable to the set of left-handed rotary operations that define $\mathcal{H}$ as a geon.

More aptly, torsion complements the TOIC's underlying principle of radical conservation of momentum. Since the goal is to connect the quantum and gravitational sectors, it must be noted that general relativity without torsion does not conserve angular momentum globally. The orbital momentum is conserved, but to conserve angular momentum including spin, i.e. all of the angular momentum, torsion is required [17].

This connection to the quantum world arises from a unique property of torsion that sets it apart from the gravitational and electromagnetic fields. Where they have scalar Coulomb and kilogram sources, the torsion field has vector sources. The source vectors could be spin vectors, the ontological vectors, the vectors that separate the $\hat{\pi}$-sites, the various arrows of time, or possibly the four new vectors $\vec{g}_{\pm\mu}$ and $\vec{g}_{\pm\nu}$. In reference [17], the source of torsion is identified as the tangent vector to a string. In that case, the sources are the wave vectors shown in figure 2a, each tangent to a string of length π.

Furthermore, the torsion in a theory can enter directly as the field or indirectly as the potential. This means the torsion can be the hyperstructure encoded on the point density functions for each of the theory's various continua.

Consider the following general description of torsion from reference [18].

> "Suppose we have two vectors A and B. They are in a plane (of course), originally with the bases touching. Here is a little thought experiment you may do. Parallel transport A along B and mark where the tip of A is. Call this trip one. Now go back and set the vectors as they were, but this time parallel transport B along A and mark the point where the tip of B is. Call this trip two. Is this the same point? In Euclidean geometry it is, and it is also the same point in Riemannian geometry, but in the richer non-Riemannian geometry (a poor name, but it means non-Riemannian and non-Euclidean) it is not the same point. In fact, the difference between these points is proportional to the torsion."

In light of that last part, torsion has direct application to the arrow of time. Let A and B be the vectors connecting the origin of $\mathcal{H}$ with those in $\Sigma^{\pm}$. Since parallel transport along one and then the other will lead to different endpoints depending on the ordering, trajectories through the torsion field in the space between adjacent moments are not symmetric under chirological time conjugation.

A standard method of comprehension is to consider the tensor transformation law. Hamiltonian physics represents a baseline, rank-two tensor field and we want to add a source and sink of information through a multiplectic expansion to rank-three. In that regard, torsion is the only reasonable candidate for integration with the existing theory so there is an *a priori* requirement for it without making reference to string theory. Transformations of the torsion tensor go as follows.

$$S^{\mu'}_{\nu'\rho'} = \frac{\partial x^{\mu'}}{\partial x^{\mu}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} \frac{\partial x^{\rho}}{\partial x^{\rho'}} S^{\mu}_{\nu\rho} \tag{80}$$

The O(3,1) structure is apparent and it is easy to apply the logic from equations (2) and (3) to define a four-fold multiplex.

$$S^{\mu'}_{\nu'\rho'} = \left(\frac{\partial x^{\mu'}}{\partial x^{\mu}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} \frac{\partial x^{\rho}}{\partial x^{\rho'}} \right) \widehat{S}^{\mu}_{\nu\rho} \tag{81}$$

$$S^{\mu'}_{\nu'\rho'} = \left(\frac{\partial x^{\mu'}}{\partial x^{\mu}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} S^{\mu}_{\nu\rho} \right) \frac{\widehat{\partial x^{\rho}}}{\partial x^{\rho'}} \tag{82}$$

$$S^{\mu'}_{\nu'\rho'} = \left(\frac{\partial x^{\mu'}}{\partial x^{\mu}} \frac{\partial x^{\rho}}{\partial x^{\rho'}} S^{\mu}_{\nu\rho} \right) \frac{\widehat{\partial x^{\nu}}}{\partial x^{\nu'}} \tag{83}$$

$$S^{\mu'}_{\nu'\rho'} = \left(\frac{\partial x^{\nu}}{\partial x^{\nu'}} \frac{\partial x^{\rho}}{\partial x^{\rho'}} S^{\mu}_{\nu\rho} \right) \frac{\widehat{\partial x^{\mu'}}}{\partial x^{\mu}} \tag{84}$$

Tensor transformations imply covariant derivatives and it is no surprise that $\alpha_{MCM} = 2^1\pi^1 + \Phi^3\pi^3$ looks like it the returned value of a covariant derivative. (The exponents are written to demonstrate α's O(3,1) character.)

Equations (81-84) are adapted to quaternions as follows.

$$v_0 \mathbf{1} \equiv \widehat{S}^{\mu}_{\nu\rho} \tag{85}$$

$$\vec{v} \equiv \frac{\widehat{\partial x^{\mu'}}}{\partial x^{\mu}} + \frac{\widehat{\partial x^{\nu}}}{\partial x^{\nu'}} + \frac{\widehat{\partial x^{\rho}}}{\partial x^{\rho'}} \tag{86}$$

These final representations may seem like unnatural invocations, but the physical characteristics of torsion are so well-suited to the related qualitative principles that they should not be discarded prematurely.

QFT

TOIC quantum cosmology treats the universe as a field excitation in a path integral framework. Note the identity map to $\hat{2}$ (such as might be associated with the creation of a computational state) appears as the kinetic energy term in the action that allows us to compute the motion.

$$\langle q_F|e^{-i\hat{H}t}|q_I\rangle = \int Dq(t')e^{i\int_0^t dt'\,\frac{1}{2}m\dot{q}^2} \tag{87}$$

Lorentz invariance is based on orientation with respect to the arrow of time, but now there is a fractal quality on ∂_t where new group operations decompose amplitudes in time, move them through a certain algorithm, and reassemble them at other points in spacetime where a physicist has an opportunity to measure something. Should one desire to compute infinitely complex equations of motion, the connection between the time derivative and the multiplex must be clarified. Can we define a gradient $\nabla^\xi : \{\partial_\pi, \partial_\Phi, \partial_2, \partial_i\}$ that replaces $\hat{1}$ in equation (13)? Is $\hat{2}$ involved in the derivative or is $\hat{2}$ only related to the prefactor $1/2$ that pops up almost everywhere in physics? Is $\hat{2}$ related – perhaps – not to one thing, but to *two*? None of these important questions are treated here. Instead, we detail how the TOIC path integral differs from the Feynman path integral. Consider the difference between equations (73) and (88).

$$\mathcal{Z}(0) = \langle 0|e^{-i\hat{H}t}|0\rangle = 1 \tag{88}$$

The probability for the particle to stay in the ground state in the absence of a source is 100% in the classical formulation but here that amplitude is zero. The state in one moment is distinct from the identical state in other moments, and it is forbidden to remain in an unchanging moment.

The TOIC also requires revision to the path integral measure Dq.

$$\int Dq(t) \equiv \lim_{N\to\infty}\left(\frac{-im}{2\pi\delta t}\right)^{\frac{N}{2}}\left(\prod_{k=1}^{N-1}\int dq_k\right) \tag{89}$$

We need to add one term $dq_\aleph$ to the product operator to account for the expanded topology $\mathbb{R}\otimes\aleph$ and this brings the upper bound on k to a more natural looking N. The factor related to the square root of i is vexing but not unexpected due to the unresolved interpretive mysteries related to the complex amplitude. If it were possible to define a better computation proportional to the square of $i^{1/2}$, that would resolve the problem by making i always appear in integer powers.

Even better, if it appeared to the fourth power, that would remove the imaginary component of the measure's prefactor altogether. This is very easy to achieve with the determinant of a matrix whose four diagonal components represent the ontological basis. Problem solved. If the determinant of that matrix has a phase shifted term associated with $\hat{i}$, it will be possible to write the measure as follows.

$$\int D'q(t) \equiv \lim_{N\to\aleph_0}\prod_{k=1}^{N}\left[\left(\frac{m}{2\pi\delta t}\right)^2\int dq_k\right] \tag{90}$$

This is more than a superficial modification because $D'q$ cannot be derived from the usual expansion of $\langle q_F|e^{-i\hat{H}t}|q_I\rangle$ by butterfly operators.

$$\langle q_F|e^{-iHt}|q_I\rangle = \left(\prod_{k=1}^{N-1}\int dq_k\right)\langle q_F|e^{-i\delta Ht}|q_{N-1}\rangle\ldots \\ \ldots\langle q_1|e^{-i\delta Ht}|q_I\rangle \tag{91}$$

When the extra term $dq_\aleph$ is added, it appears as an unpaired term at both ends, in perfect keeping with the over-arching principles of the TOIC.

$$\langle q_F;\pi_1|e^{-iHt}|q_I;\pi_0\rangle = \\ = \left(\prod_{k=1}^{N}\int dq_k\right)|q_\aleph\rangle\langle q_F|e^{-i\delta Ht}|q_{N-1}\rangle\ldots \\ \ldots\langle q_1|e^{-i\delta Ht}|q_I\rangle\langle q_\aleph| \tag{92}$$

The classical path integral is discarded. Instead, we are saying the formulation in equation (90) is the correct framework in which to model multiplexed wavepackets with different arrows of time in the past, present, and future.

DEVELOPMENTS AT LARGE

The goal is to use math to engineer longitudinal physics which can be used to predict the outcome of experiments in the future. In addition to the developments in nuclear fusion detailed in reference [6], some other recent developments in empiricism seem equally connected to present matters.

Time reversal symmetry violation In reference [7] we hypothesized that due to discrepancies between the past and future, it might be possible to measure variations in the fine structure constant by varying the delay in an appropriate experiment. Just months after our prediction, BaBar's dataset was reanalyzed for correlations with delay. They did find one and published their fantastic results in reference [19].

Recall the genius of the Hamiltonian formulation: once any two dynamical variables are known, everything is known (pretty much). Therefore a function defined on the plane spanned by two such variables, such as those in figure 4, can be used to deduce the fine structure constant. Since the four fitting functions in figure 4 are different, the value for α does depend on the delay.

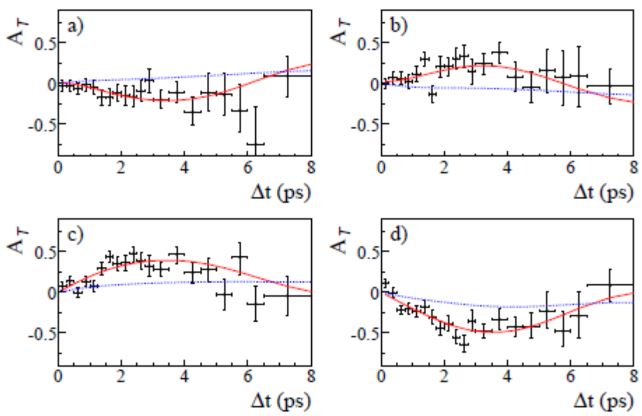

FIG. 4. Four independent T-violating symmetries from reference [19]. "The decays of entangled neutral B mesons into definite flavor states (B^0 or $\bar{B}^0$), and $J/\psi K_L^0$ or $c\bar{c}K_S^0$ final states (referred to as B_+ or B_-),allow comparisons between the probabilities of four pairs of T-conjugated transitions, for example, $\bar{B}^0 \to B_-$ and $B_- \to \bar{B}_0$, as a function of the time difference between the two B decays."

Negative frequency resonant radiation The physical basis of the MCM is that momentum should always be conserved. That led to the inescapable conclusion that there must be a cosmological component unfolding in negative time. Reference [20] describes the discovery of negative frequency optical modes and figure 5 shows the new peak. Since frequency is inverse time, this must be the same physical principle at work. Consider what the discoverers point out in reference [21].

> "The momentum conservation law that governs the scattering process predicts that light may resonantly scatter into two output modes [resonant radiation] and [negative frequency resonant radiation]. In the laboratory reference frame, both of these modes will have positive frequencies while in the reference frame comoving with the scatterer, RR is positive and NRR is negative valued."

The main result of reference [21] was that a relativistic inhomogeneity (RI) propagating in an optical medium will output more photons than are input as a result of the interactions between the positive and negative frequency modes. They even go so far as to describe the RI as "the analogue of an event horizon." This system seems quite similar to the shear plane $\mathcal{H}$ that is our cosmological inhomogeneity. In fact, if more photons come out than go in, that suggests increasing information and their equation for the process may be a perturbative analogue of $\Phi - \varphi = 1$.

$$|RR|^2 - |NRR|^2 = 1 \tag{93}$$

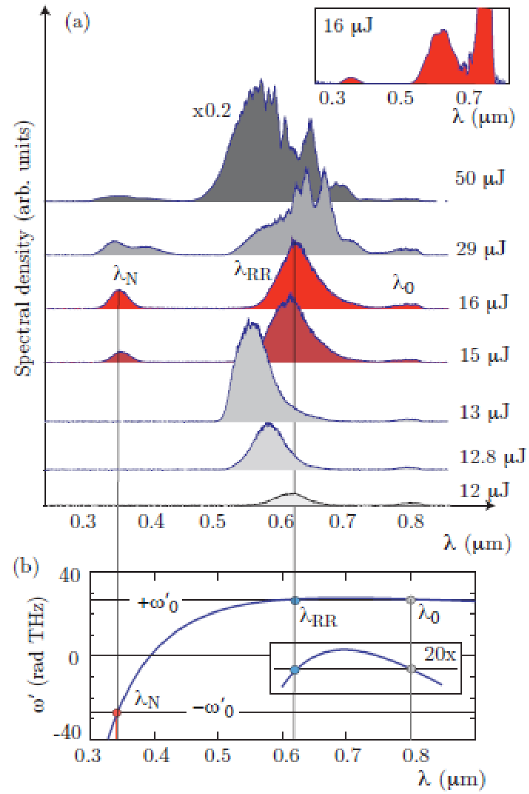

FIG. 5. The mode labeled λ_N in figure 5a is a newly discovered negative frequency mode, as shown in figure 5b. Figure excerpted from reference [20]

THE END

Where chiros "originally" is a Greek word meaning God's time, here it can be understood as the computational time and the following elephant in the room gives a congruence for orthogonality in the transverse and longitudinal phases.

$$\Phi \approx 1.62 \tag{94}$$

$$\frac{\pi}{2} \approx 1.57 \tag{95}$$

$$\Delta\aleph \approx 3\% \tag{96}$$

$$\frac{\pi}{2} \cong i\Phi \tag{97}$$

In the regime of physicists in real life, it is possible to use the following meso-scale equation.

$$2\Phi = \pi \tag{98}$$

It's true that the relative magnitude of the part that is wrong is very large, but so what? The magnitude of the error always ends up being arbitrary anyway; that is physics' divergence from mathematics. Physical equations always become approximations so the equals sign normally means "almost equal" and that is what it still means in equation (98).

In machine precision, it is impossible to simulate the complete spectrum of the Cantor flux of the information current [4]. It will be precisely in those tails that are cropped that one finds the ultra-high energy qubits with which it is supremely easy to initiate acceleration, jerk, etc, on the scale of the dynamics. By adding a term at infinity, it is always possible to calculate spline functions which can be sampled to give a fair representation of the unbounded spectrum of information. With only a pair of points in the past and present *à la* old physics, it is impossible to calculate a cubic spline.

The initial connection of the MCM to general relativity arose in the map from frequency to angular frequency [5] so consider the following novelty.

$$\hat{1} \equiv 1^f = (e^{2\pi i})^f = e^{2\pi i f} = e^{i\omega} \tag{99}$$

U(1) states are vectors in Hilbert space that look like $e^{i\omega}$, so it does seem like a logical connection that any generic state can be written as a unit vector. Comparing to the identities in equation (56), we see that every single object at every level of the theory can be represented as unity though a judicious choice of gauge.

To finish with an interesting calculation, let $\hat{\boldsymbol{o}}$ be a quaternion rotation operator related to π. Let λ be a coupling constant in the future dual to the fine structure constant α in the past, where all measurements were made. Let the phases in the past and future go as $i\varphi$ and $i\Phi$ respectively. We want to create a discontinuity in the present by utilizing an asymmetry between the Φ-based quaternion rotations on the left and right. All the required physics is encapsulated in the following equation.

$$\psi' = (i\Phi\hat{\lambda})(\hat{\boldsymbol{o}}_L \psi \hat{\boldsymbol{o}}_R)(i\varphi\hat{\alpha}) = \Phi i \lambda \boldsymbol{o} \psi \boldsymbol{o} \varphi i \alpha \tag{100}$$

[1] J. Tooker, ***Dark Energy in M-Theory***, viXra:1208.0077 (2011)

[2] R. Arnowitt, S. Deser, and C.W. Misner, ***Note on the Positive-definiteness of the Energy of the Gravitational Field*** Annals of Physics, 11, 116 (1960)

[3] J. Tooker, ***Geometric Cosmology***, viXra:1301.0032 (2013)

[4] J. Tooker, ***Ontological Physics***, viXra:1312.0168 (2013)

[5] J. Tooker, ***Tempus Edax Rerum***, viXra:1209.0010 (2012)

[6] J. Tooker, ***Quantum Gravity***, *citation pending* (2015)

[7] J. Tooker, ***Derivation of the Fine Structure Constant***, viXra:1208.0076 (2012)

[8] Alfred D. Shapere, Frank Wilczek, and Zhaoxi Xiong, ***Models of Topology Change***, hep-th/1210.3545 (2012)

[9] M. Asorey, A. Ibort, and G. Marmo, ***Global Theory of Quantum Boundary Conditions and Topology Change***, hep-th/0403048 (2005)

[10] A. P. Balachandran, G. Bimonte, G. Marmo, and A. Simoni, ***Topology Change and Quantum Physics***, gr-qc/9503046 (1995)

[11] J. Tooker, ***Modified Spacetime Geometry Addresses Dark Energy, Penrose's Entropy Dilemma, Baryon Asymmetry, Inflation, and Matter Anisotropy***, viXra:1302.0022 (2009)

[12] Abhay Ashtekar and Parampreet Singh, ***Loop Quantum Cosmology: A Status Report***, Class. Quant. Grav. **28** (2011)

[13] Hans C. Ohanian, ***What is Spin?***, Am. J. Phys, Vol. 54, No. 6 (1986)

[14] J. Tooker, ***Quantum Structure***, viXra:1302.0037 (2013)

[15] J.M. Overduin and P.S. Wesson, ***Kaluza-Klein Gravity***, gr-qc/9805018 (1998)

[16] J. Tooker, ***Kerr-Newman, Jung, and the Modified Cosmological Model*** viXra:1405:0329 (2014)

[17] Richard T. Hammond, ***New Fields in General Relativity***, Contemp. Phys., vol. 35, no. 2 (1995)

[18] Richard T. Hammond, ***Spin, The Classical Theory***, J. Mod. Phys., 3 (2012)

[19] The BABAR Collaboration, ***Observation of Time Reversal Symmetry Violation in the B0 Meson System***, arXiv:1207.5832v1 (2012)

[20] E. Rubino *et al.*, ***Negative Frequency Resonant Radiation***, arXiv:1201:2689 (2012)

[21] E. Rubino *et al.*, ***Soliton Induced Relativistic-Scattering and Amplification***, arXiv:1211:0256 (2012)